

## THINK OUTSIDE THE SQUARE WITH SUPER RULE™

### Preparation:

Your **SUPER RULE™** is a valuable acquisition for classroom and home use. Mark it with your name in permanent ink and store in a ring-binder file. (Note the small holes on the left side.)

It is made of a quality "soft feel" material, which ensures it will be extremely strong and flexible. It is also coloured light green for easy viewing on desk or in hand, night or day.

The shapes are designed for accurate drawing with a **1mm diameter** pencil lead or biro point. That is, if you draw around the inside of a shape such as a circle, the final drawn line will be **0.5mm** inside the template cut-out (all the way around). This means that the cut-out for a 40mm square (say) is actually 41mm, so that when the shape is drawn it will finish at the required 40mm. You will find that most "medium" point pens or pencils will do the job.

You should experiment with your pens and pencils to find those that "best fit".

To test your pen/pencil for suitability, try superimposing some related shapes (such as the square and the circle in Cluster B, below). If the shapes fit each other exactly, once drawn, then you have a suitable pen.

### About the Template:

The functions of the **SUPER RULE™** can be described as either:

- **primary functions** – straight forward measuring with the linear or circular scales, or drawing simple shapes with one of the template cut-outs provided.
- **secondary functions** – where two or more shapes are combined to draw a more complex shape.

The **SUPER RULE™** has been especially designed to enable a large number of primary and secondary functions to be achieved. Your template is a world leader in this regard.

## About the Shapes:

All the common shapes are there for your use. Circles, ellipses, squares, rectangles, triangles (equilateral, isosceles, scalene; right, right isosceles), parallelograms, rhombuses, a trapezium, a regular hexagon, a regular pentagon and a regular octagon.

Different rectangles, squares, triangles and trapeziums can also be drawn by connecting suitable vertices of the regular octagon, hexagon and pentagon. (Have a go – there's many possible.)

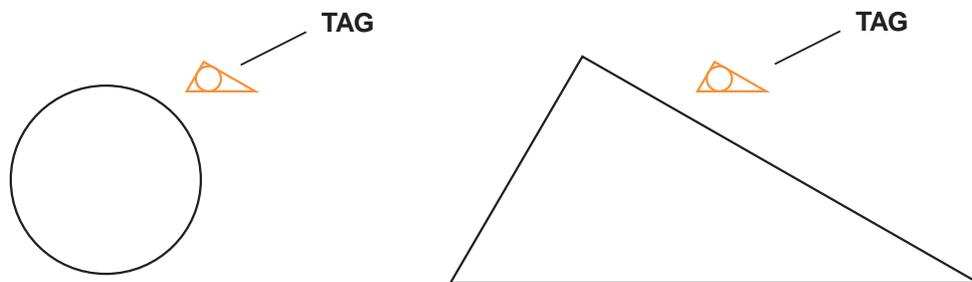
If you wish to locate the centre of a circle once it has been drawn, use the special arc graduations located on the vertical and horizontal axes of the hyperbola.

The protractor can be used to accurately measure and draw angles, both in degrees and radians (1 radian is approximately  $57.3^\circ$ ;  $\pi$  radians is exactly equal to  $180^\circ$ .)

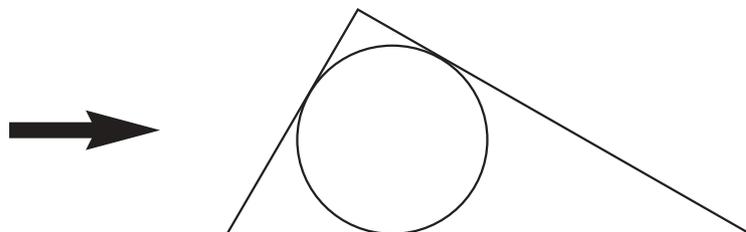
## About TAGs™:

The unique Template Assistance Graphics are provided to make some of the secondary drawing easier. When certain shapes are related so that one fits inside (or outside) of the other (this is called inscribing or circumscribing, respectively), TAGs are shown alongside the cut-out so that you can see at-a-glance which shape goes with which.

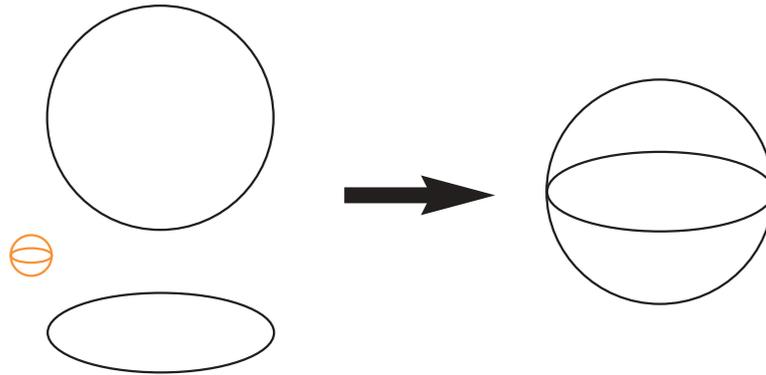
As an example, note the twin TAGs alongside these two shapes:



Hence this drawing results, combining both shapes, of a circle inscribed within a right triangle:



When there is only a single TAG (no twin), the TAG is telling you that the two shapes either side of it are related. For example, this pair of shapes and the TAG can be used to inscribe the ellipse inside the circle:

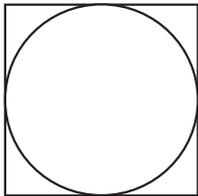
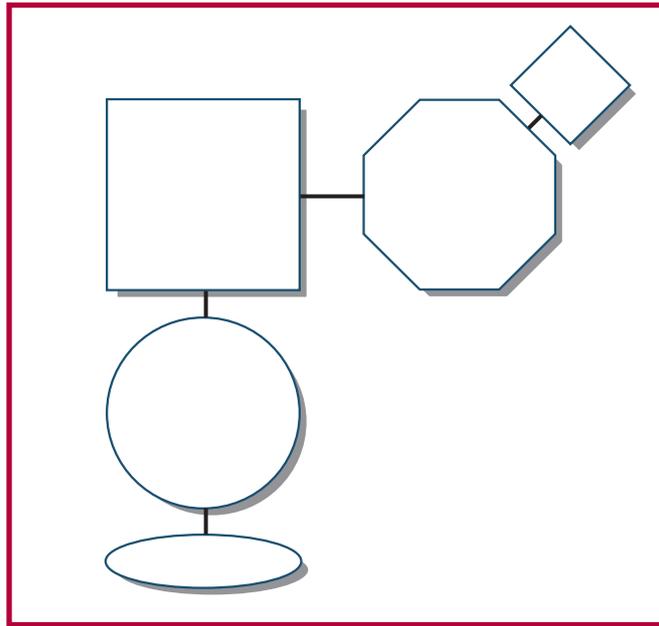


### About Clusters:

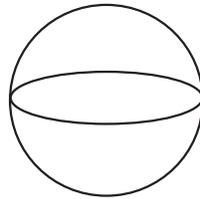
Examine your **SUPER RULE™** carefully and you will see that many of the shapes are connected by thick lines. Such related shapes are arranged in **clusters** (top left of the template, top right of the template, etc.) and can be drawn together to create lots of important diagrams. Some shapes have both lines connecting them and TAGs. This is just telling you in two ways that the shapes are related and that secondary drawing is possible.

The following shapes are examples of what can be done with the clustered shapes. They serve only as examples – there are many other possibilities.

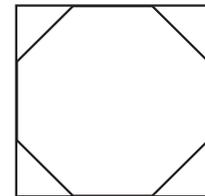
**CLUSTER A** (Top Left):



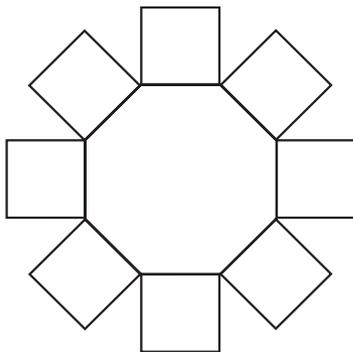
**Circle inscribed within a square.**



**Ellipse inscribed within a circle to create a 3-D spherical effect.**

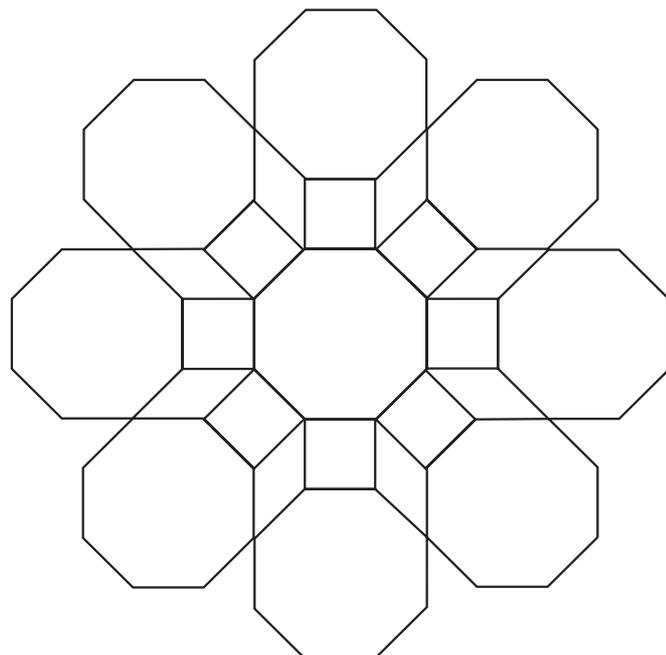


**Octagon inscribed within a square.**

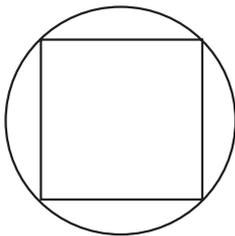
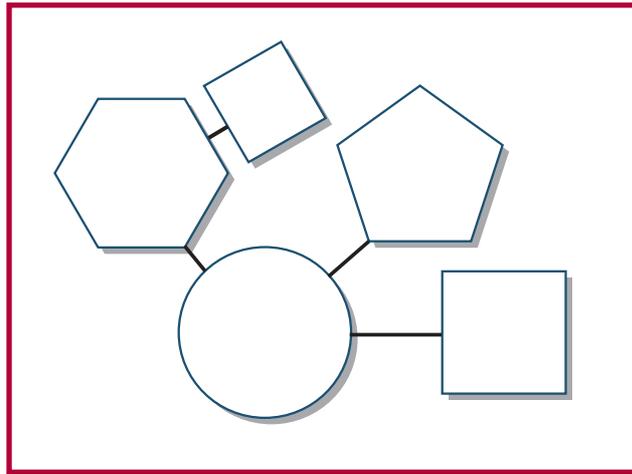


**Squares drawn to the sides of the regular octagon.**

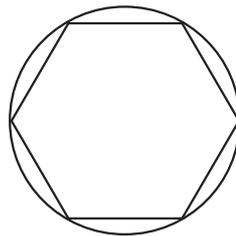
And if the drawing is continued, a **tessellation** like that to the right results. Are there any others?



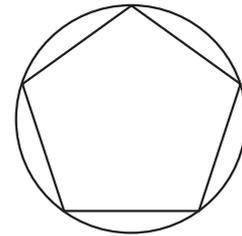
**CLUSTER B** (Top Right):



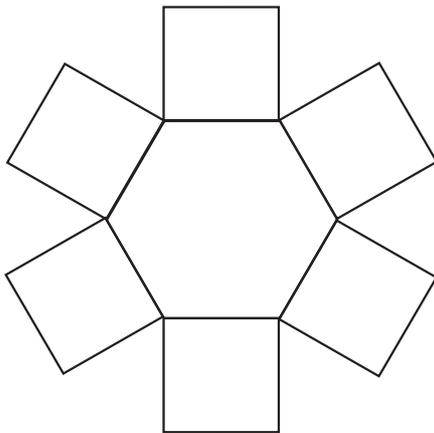
**Circle circumscribing a square.**



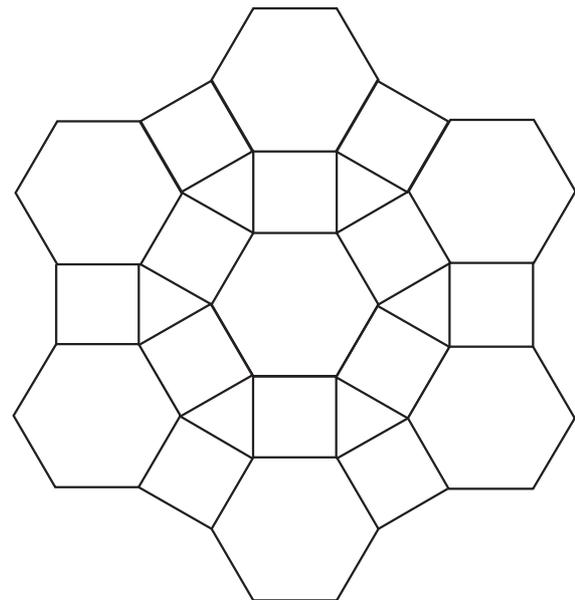
**Circle circumscribing a regular hexagon.**  
This circle can also be used to circumscribe an equilateral triangle, a right triangle, a rectangle and others. (Just connect suitable vertices of the regular hexagon.)



**Circle circumscribing a regular pentagon.**  
This circle can also be used to circumscribe isosceles triangles and a trapezium. (Just connect suitable vertices of the regular pentagon.)

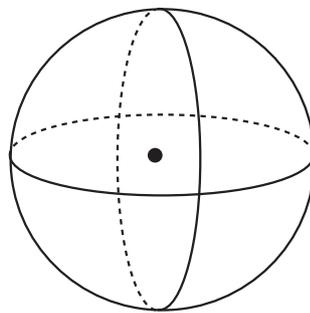
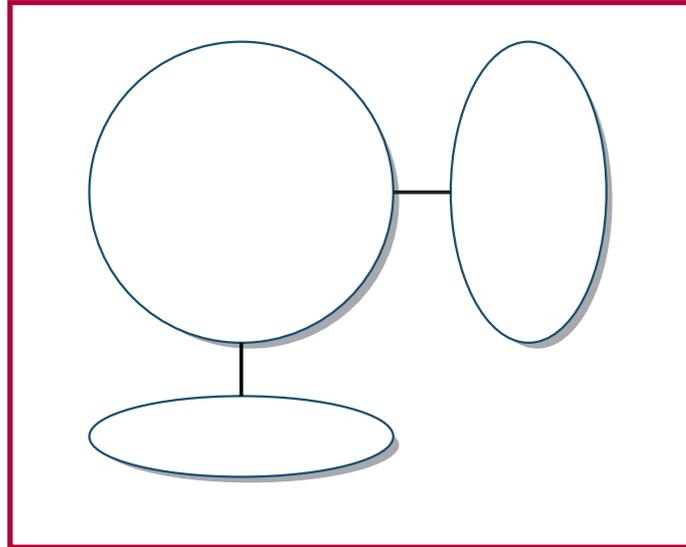


**Squares drawn to the sides of the regular hexagon.**

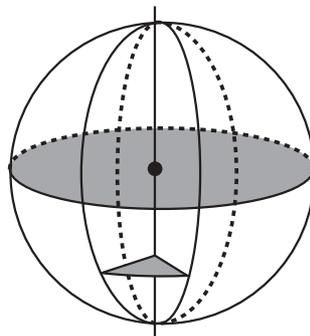


And if the drawing is continued, a tessellation like that to the right results. Are there any others?

**CLUSTER C** (Middle Left):

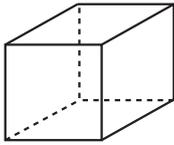


**Ellipse drawn twice within a circle.**  
(The horizontal ellipse is rotated  $90^\circ$  anticlockwise about the centre.)



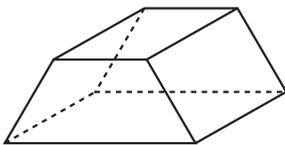
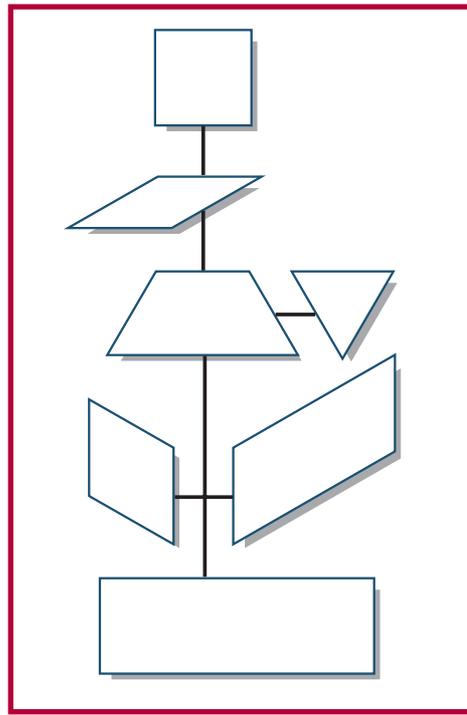
When both the wider and narrow ellipses are used, more detailed diagrams are possible. These are very useful when drawing **lines of longitude and latitude** around a globe.

**CLUSTER D** (Bottom Left):



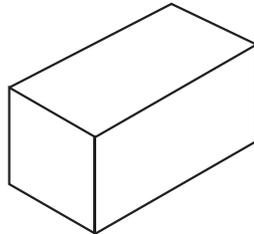
**Cube**

Just draw the square twice (translate the first shape) and join corresponding vertices.



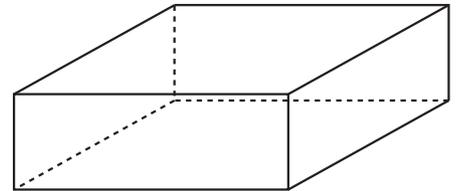
**Trapezial Prism**

Draw the trapezium twice (translation) and join corresponding vertices.



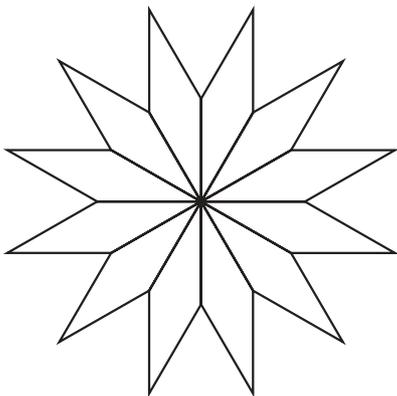
**Square Prism**

This version is drawn as a solid without the "transparent" appearance of those previous. It can be drawn by translating the small rhombus or drawing both the small rhombus and the adjacent parallelogram.

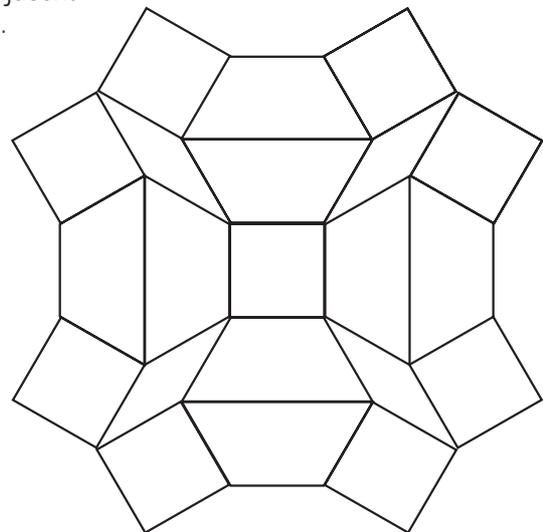


**Cuboid or Rectangular Prism**

Can be drawn by either translating the parallelogram or by translating the rectangle.

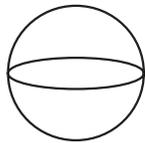
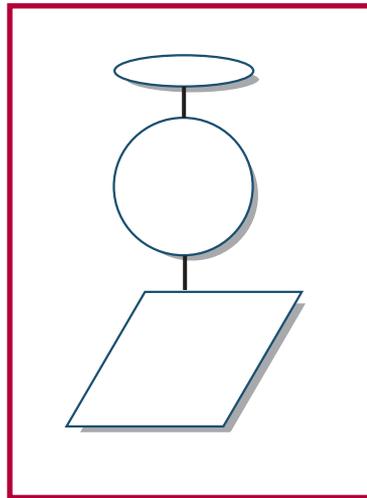


**Patterns** like these can be created by rotating a suitable rhombus. Not all rhombuses will tessellate like this. Why not?

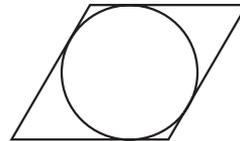


There are lots of tessellations possible using the shapes of Cluster D. This is just one example.

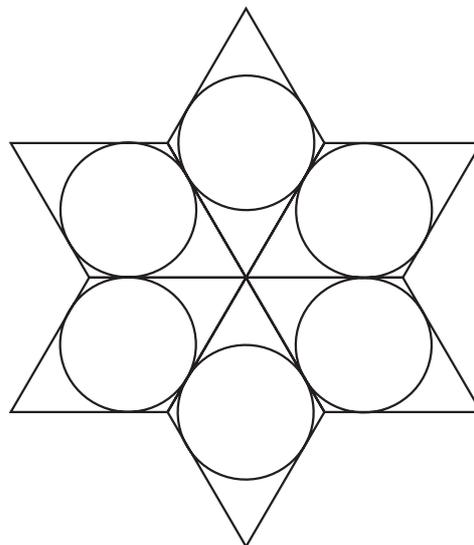
**CLUSTER E** (Bottom Right):



**Ellipse inside the  
small circle.**

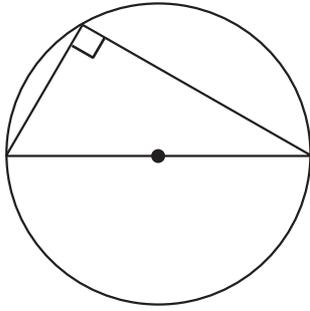


**Circle inscribed within  
the rhombus.**

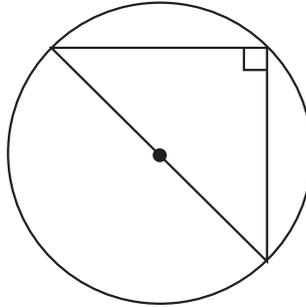


**Another tessellating pattern.**  
Others are possible.

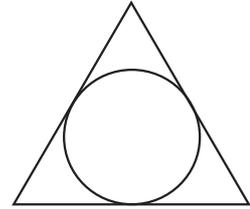
MISCELLANEOUS RELATED SHAPES:



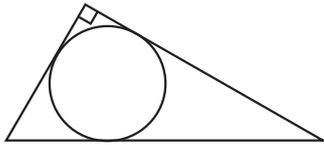
The 1, 2,  $\sqrt{3}$  ( $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ ) **right triangle** is circumscribed by the large circle. Note its hypotenuse becomes a diameter. Note also that the relationship between the circle and the right triangle is indicated on the **SUPER RULE™** with twin TAGs.



The 1, 1,  $\sqrt{2}$  ( $45^\circ$ ,  $45^\circ$ ,  $90^\circ$ ) **right triangle** is circumscribed by the large circle. Note its hypotenuse is also a diameter. Note also that the relationship is described with twin TAGs.



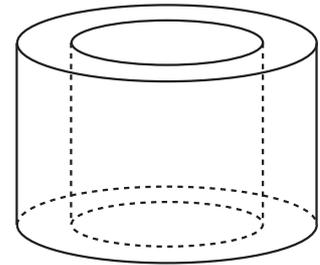
The circle near the bottom right (with cardinal points) can be inscribed within the equilateral triangle (see twin TAGs).



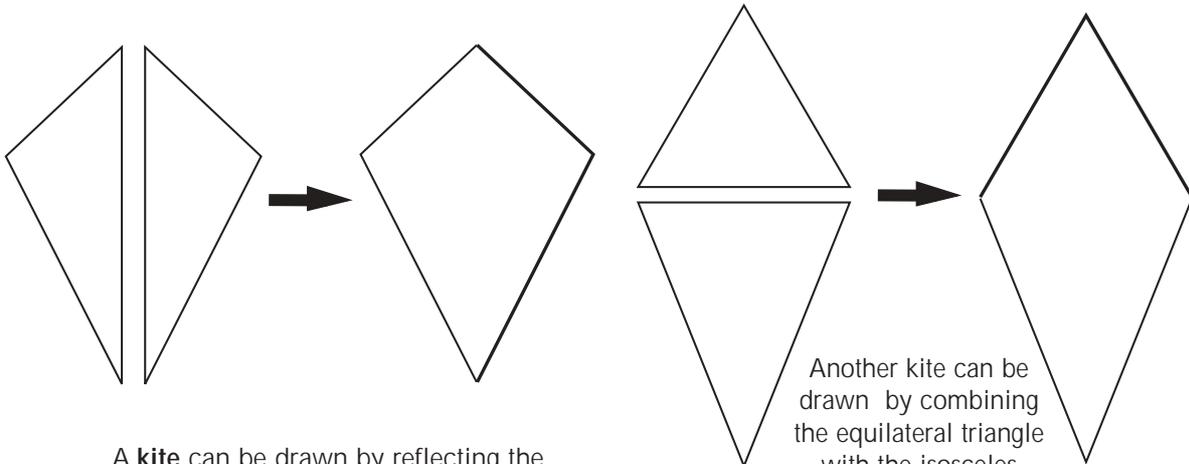
The circle near the origin of the hyperbola axes can be inscribed within the 1, 2,  $\sqrt{3}$  ( $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ ) right triangle (see twin TAGs).



The circle near the origin of the hyperbola axes can also be inscribed within the **scalene triangle** (lower right of template), see twin TAGs.



Cylinders or hollow cylinders can be drawn with selected ellipses. This diagram uses the bottom ellipses of clusters A and C.



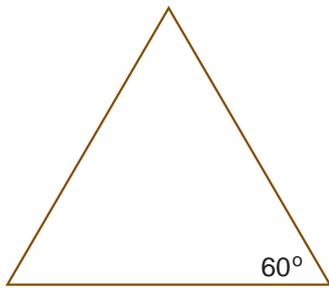
A **kite** can be drawn by reflecting the scalene triangle (above).

Another kite can be drawn by combining the equilateral triangle with the isosceles triangle below it.

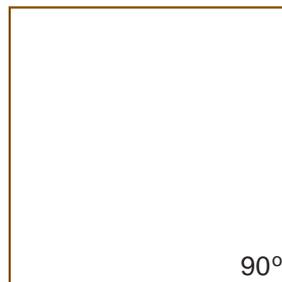
## POLYGONS and POLYHEDRA:

A polygon is a multi-sided (closed) planar figure. If all of the sides are of equal length and all angles are equal, then it is called a regular polygon.

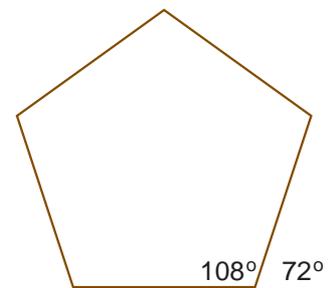
Some of the common regular polygons are:



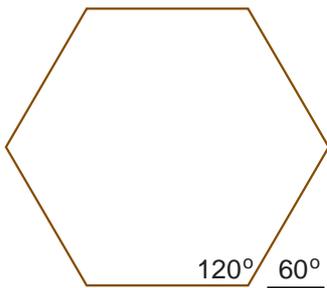
**Regular Triangle (Equilateral)**  
3 sides



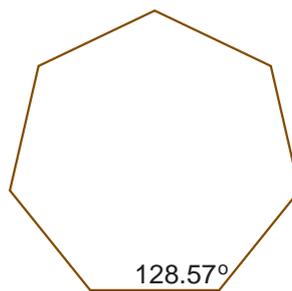
**Regular Quadrilateral – Square**  
4 sides



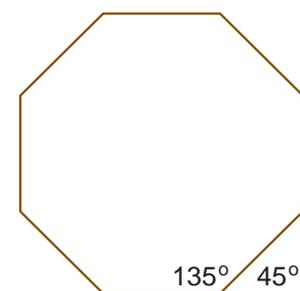
**Regular Pentagon**  
5 sides



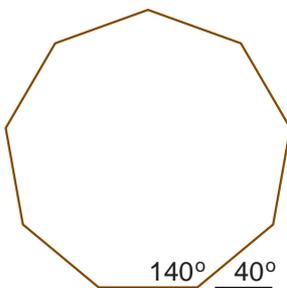
**Regular Hexagon**  
6 sides



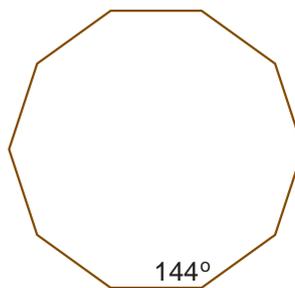
**Regular Heptagon**  
7 sides



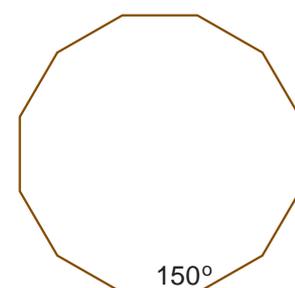
**Regular Octagon**  
8 sides



**Regular Nonagon**  
9 sides



**Regular Decagon**  
10 sides



**Regular Dodecagon**  
12 sides

The more common of the above shapes are available as cut-outs on the **SUPER RULE™**. If not, use the protractor to accurately draw them.

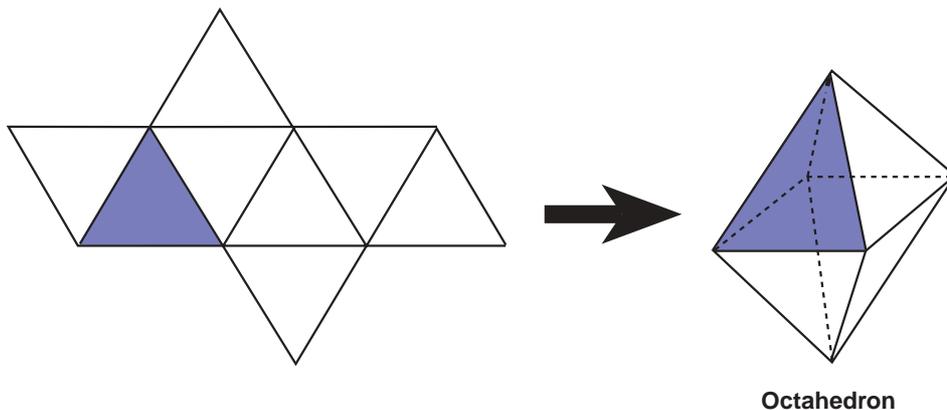
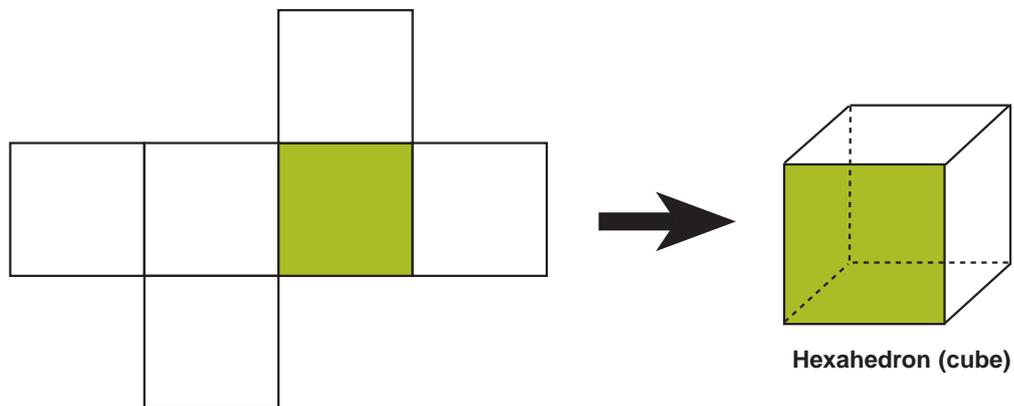
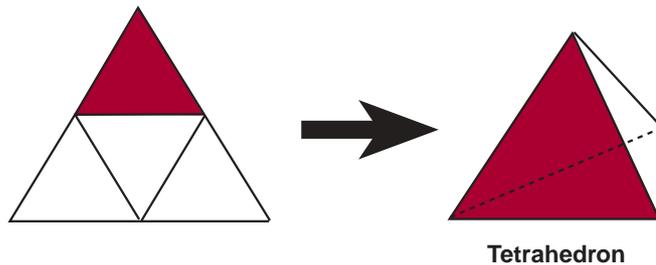
## Polyhedra:

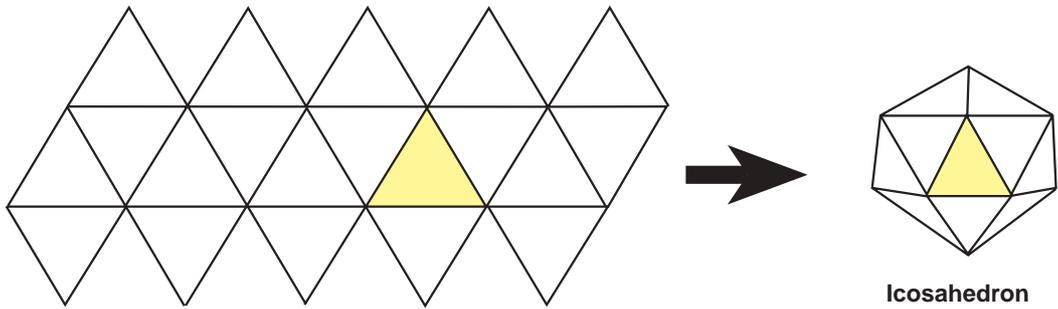
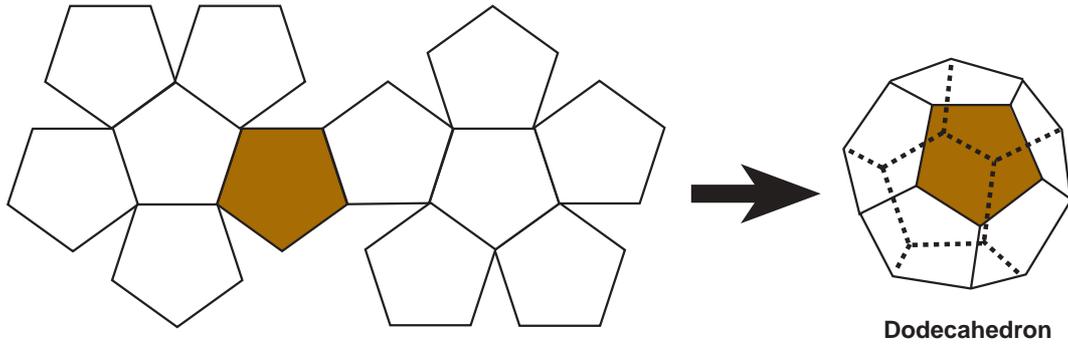
A **polyhedron** is a three-dimensional solid bounded by four or more plane faces, where each plane face is a polygon. (The plural of polyhedron is *polyhedra*.)

A **regular polyhedron** has all faces which are equal sized regular polygons, and has the same number of edges intersecting at each vertex (corner point).

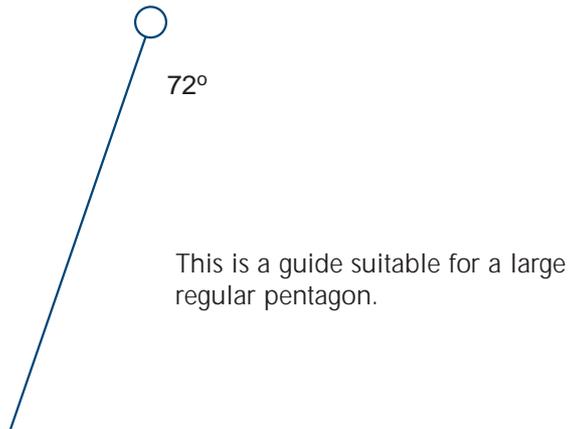
There are five regular polyhedra.

These can be constructed by drawing and then cutting out and folding appropriate **nets** of the polyhedra. In the illustrations below, the net is on the left, the regular polyhedron on the right.





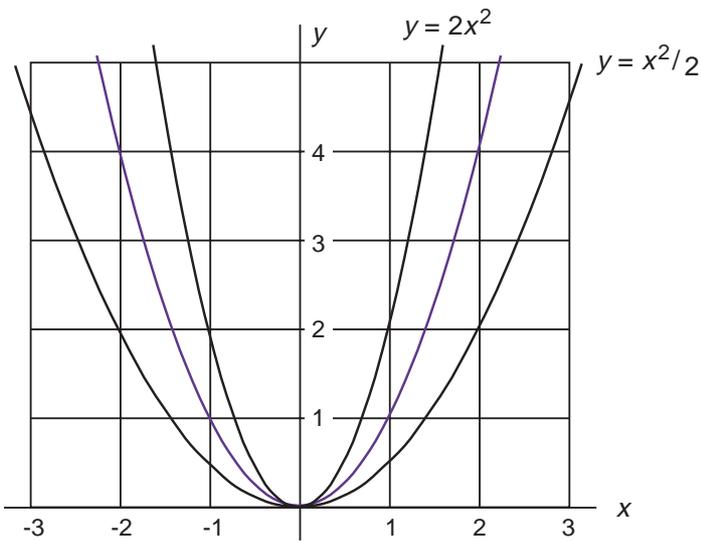
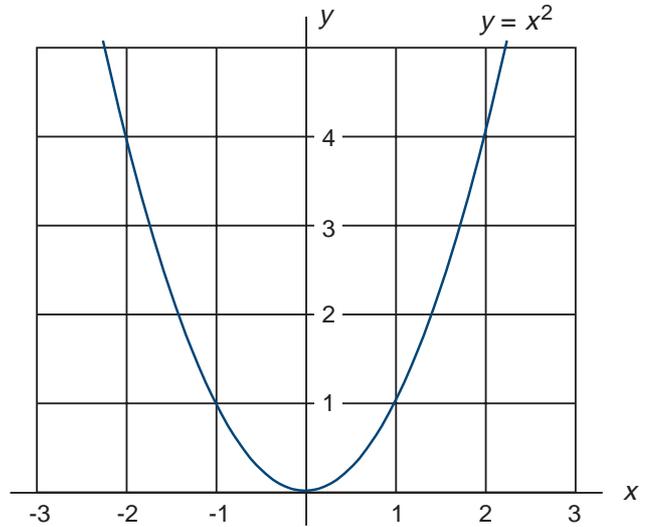
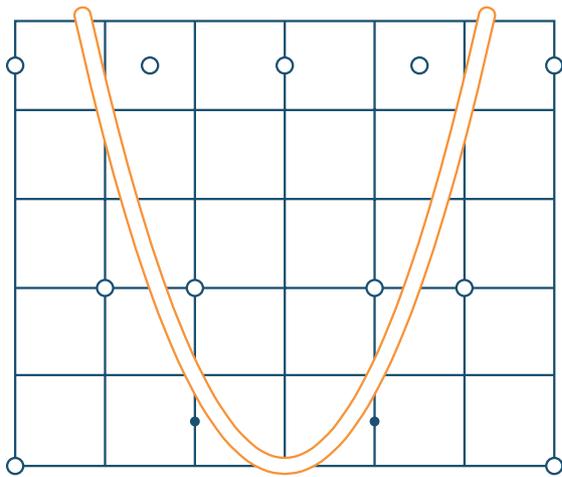
The **SUPER RULE™** has guide lines for accurately drawing giant nets. These guide lines can be found coming up from and above the protractor.



There are guides like that above available also for  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$ .

## PARABOLAS and HYPERBOLAS:

For students of algebra and calculus, some common parabolas and hyperbolas can be easily drawn.

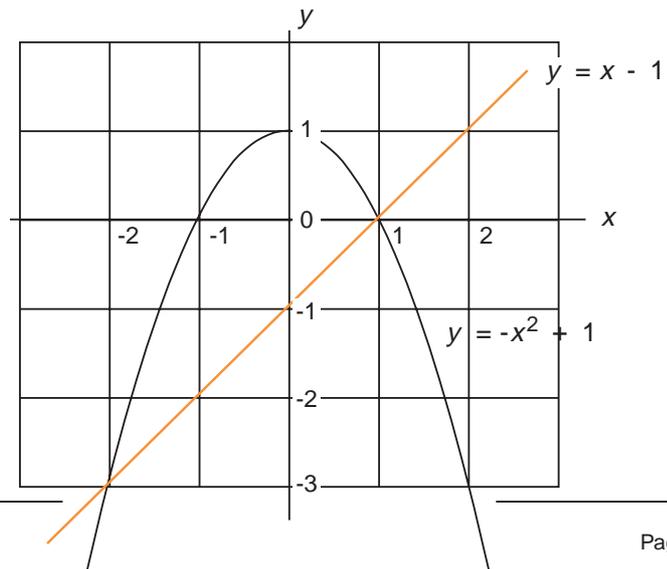


Use the small circular holes to plot the  $y = x^2/2$  and  $y = 2x^2$  curves. The small black dots are there for you to check your accuracy.

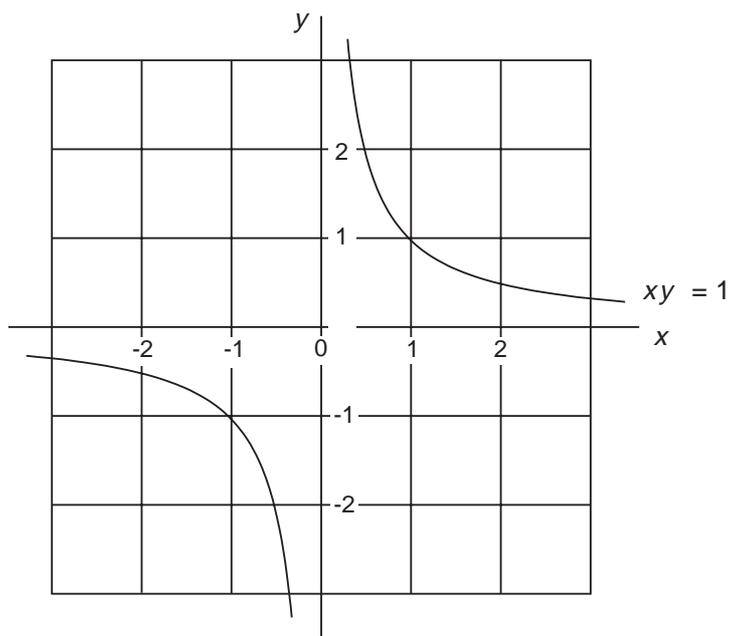
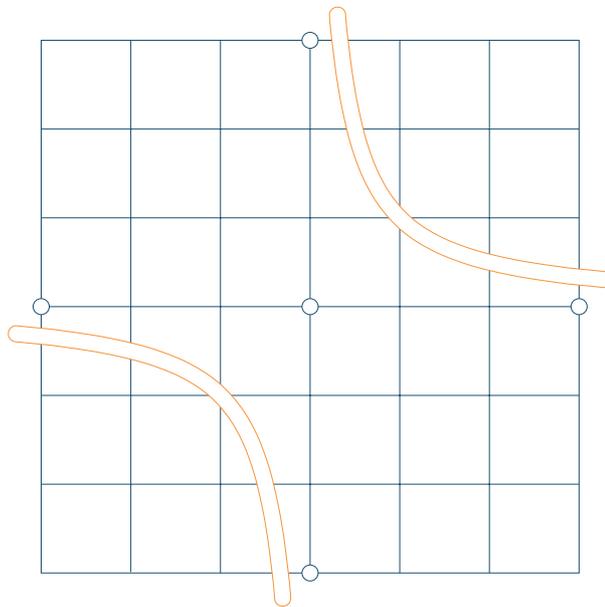
Other parabolas can be drawn by re-scaling one or both axes.

### Solution of two equations, two unknowns.

Here the simultaneous solution is (1,0) or (-2,-3).

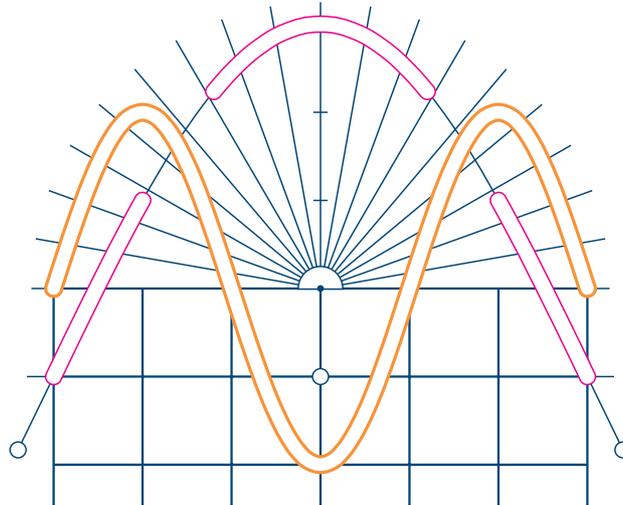


Simple **hyperbolas** can be drawn with the **SUPER RULE™**.



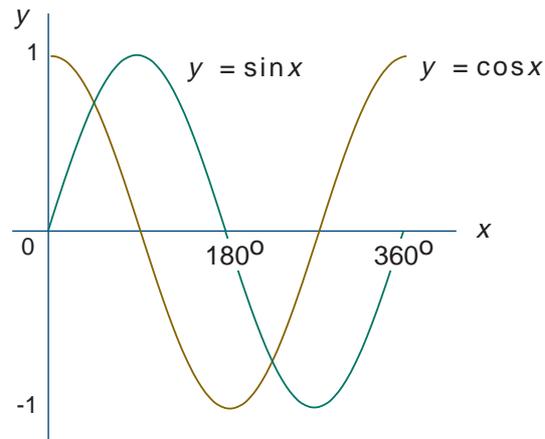
## TRIGONOMETRIC FUNCTIONS:

The shapes within the protractor can be used to draw a variety of sine and cosine curves:



The cut-outs allow you to draw sine and cosine curves with actual physical half-heights of 20mm and 40mm. The smaller curve can be extended by translating horizontally. The larger curve can be extended by rotating  $180^\circ$  about an end point (and then translating both parts, as appropriate).

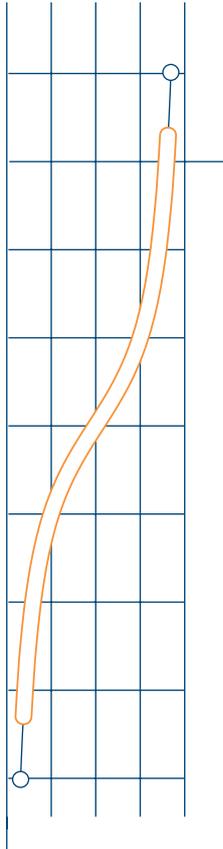
Here are two standard curves,  $y = \sin x$  and  $y = \cos x$ , both sketched for  $x$  between  $0^\circ$  and  $360^\circ$ .



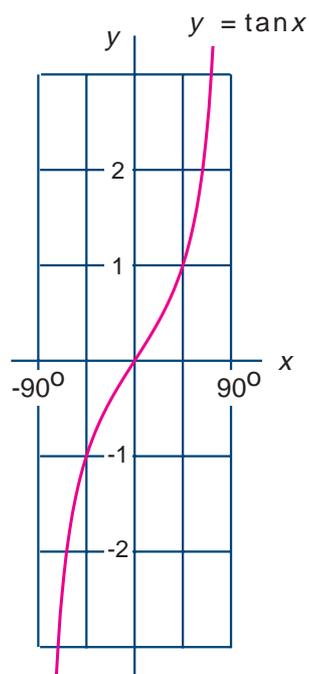
More advanced students of trigonometry can sketch curves like  $y = \sin 2x$  and  $y = 2\cos x$  by adjusting the scales on the axes.

In fact you can use either of the sine/cosine cut-outs to draw any sine/cosine curve but you will, in most cases, need to adjust the scales to suit.

There is also a tangent curve:

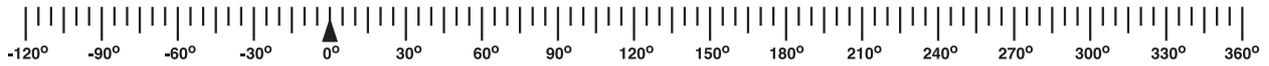


This allows for accurate sketches of tangent curves like  $y = \tan x$ , drawn here for  $x$  between  $-90^\circ$  and  $90^\circ$ . The curve reaches infinite heights at  $-90^\circ$  and  $90^\circ$  (and every  $180^\circ$  thereafter) and the vertical grid lines which will never be crossed are called *asymptotes*.

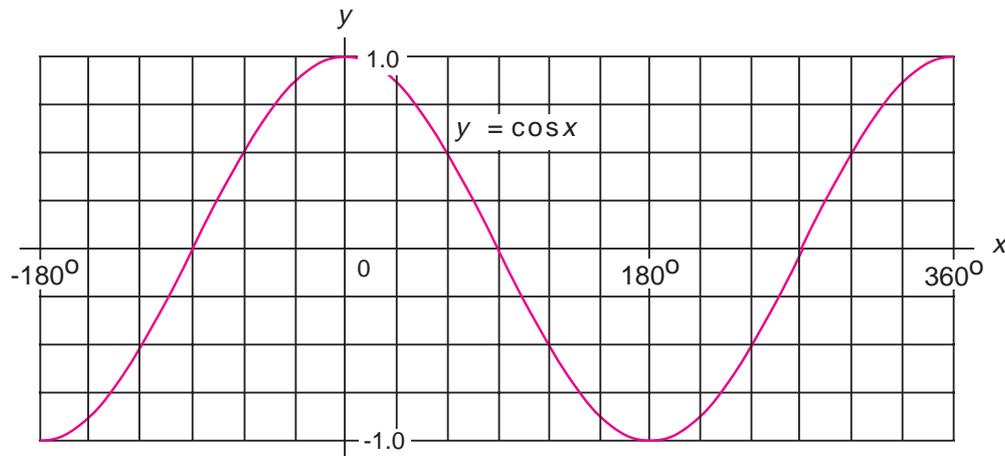


## DEGREE AND LINEAR/RADIAN SCALES and GRAPHING TRIGONOMETRIC FUNCTIONS:

The **degree axis** (located at the base of the **SUPER RULE™**):

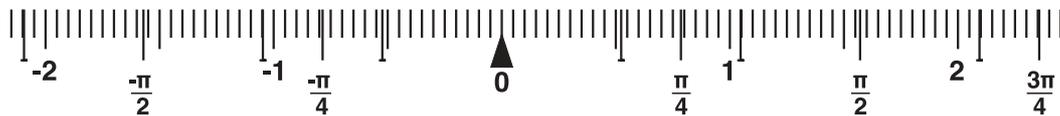


The axis can be used to graph trigonometric functions with a degree domain. Notice that it is graduated into 5° intervals (six graduations per major 30° interval). This is always suitable for freehand sketches but is sometimes exactly compatible with the cut-outs. Otherwise, it can be used with the cut-outs by changing the numerical scale to suit.



The **linear/radian axis** (located on the left of the **SUPER RULE™**):

Note that the longer graduations represent fractions of  $\pi$  (as marked). The inverted "T" graduations represent  $\pi/6$ ,  $\pi/3$ , etc., that is incrementing in  $\pi/6$  amounts.



This scale can assist you to graph trigonometric functions with a real number/radian domain, although the curve will need to be drawn freehand if you use this exact scale.

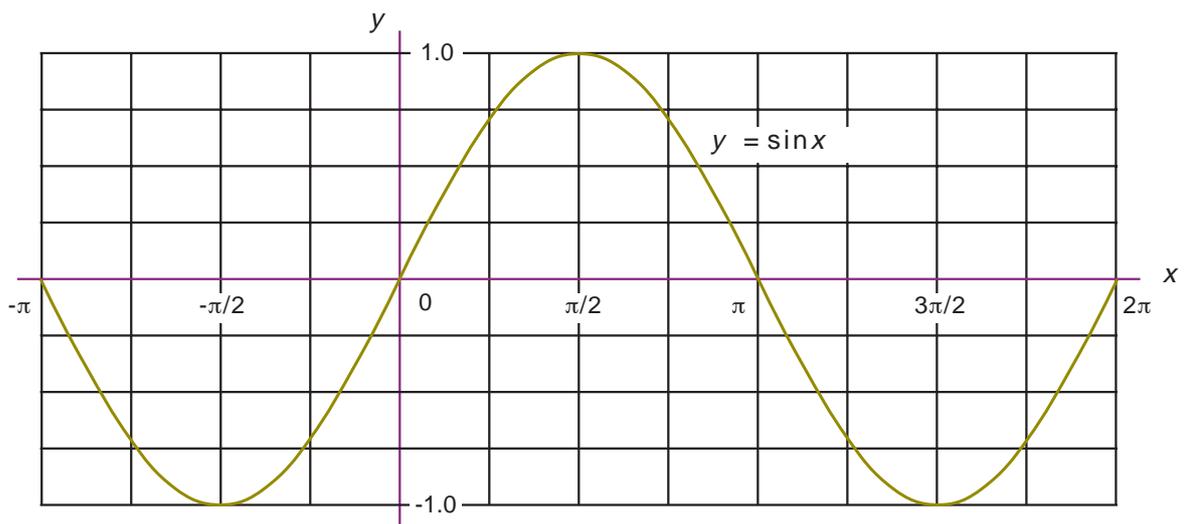
Keep in mind that  $\pi$  radians equals  $180^\circ$ .

Hence:

$$\sin \pi = \sin 180^\circ = 0$$

$$\sin \pi/6 = \sin 30^\circ = 0.5$$

$$\cos 2\pi/3 = \cos 120^\circ = -0.5, \text{ etc.}$$



*Have fun in mathematics with your **SUPER RULE™** !*